Trigonometric Functions (tangent)

The Tangent Ratio:

1) Draw segments $\overline{AB}$ and $\overline{BC}$ to form $\triangle ABC$, with $m \angle C = 90^\circ$.

2) Draw segments $\overline{C'B'}$ & $\overline{C''B''}$ such that $B'$ & $B''$ are on $\overline{AB}$, and that $C'$ & $C''$ are on $\overline{AC}$, and that $\overline{C'B'} \parallel \overline{C''B''} \parallel \overline{CB}$.

3) a) What principle proves that these triangles will be similar?

b) What is true of the parts of these triangles, given that they are similar?

4) Measure each segment

\[
\begin{align*}
AB' &= \underline{\phantom{10}} & AC' &= \underline{\phantom{10}} & B'C' &= \underline{\phantom{10}} \\
AB'' &= \underline{\phantom{10}} & AC'' &= \underline{\phantom{10}} & B''C'' &= \underline{\phantom{10}} \\
AB &= \underline{\phantom{10}} & AC &= \underline{\phantom{10}} & BC &= \underline{\phantom{10}}
\end{align*}
\]

5) Measure $\angle A = \underline{\phantom{90}}^\circ$

6) Find the following ratios: \[
\begin{align*}
\frac{B'C'}{AC'} &= \underline{\phantom{0.1}} \approx \frac{B''C''}{AC''} &= \underline{\phantom{0.1}} \approx \frac{BC}{AC} &= \underline{\phantom{0.1}} \approx \\
\text{on the calculator} & \quad \tan \left(\frac{m \angle A}{(m \angle A)}\right) \approx \underline{\phantom{0.1}}
\end{align*}
\]

**Tangent:** ratio of the lengths of the opposite side over the adjacent side, in a right triangle.

\[
\tan A = \underline{\phantom{0.000}}
\]
7) Find the tangent ratio of the acute angles in triangle MNP below,

\[
\text{written as a fraction: } \tan P = \quad \tan N = \\
\text{written as a decimal: } \tan P = \quad \tan N = \\
\text{as the result of the function: } \tan P = \quad \tan N = \\
\]

8) Using the tangent ratio to solve for missing sides and angles.

9) Practice. Solve for \( x \).
The Sine & Cosine Ratios:

10) The sine of an angle is the ratio of the lengths of the opposite side over the hypotenuse. Using the measures of the lengths of the sides in $\triangle DEF$ & $\triangle DGH$, confirm that the sine of $\angle D$ is the same for both triangles. What does your calculator compute for $\sin D$?

11) The cosine of an angle is the ratio of the lengths of the adjacent side over the hypotenuse. Confirm these ratios are consistent in both triangles for $\angle D$ above.

12) Find the following: $\tan^{-1}(\frac{EF}{ED}) = \sin^{-1}(\frac{EF}{DF}) = \cos^{-1}(\frac{DE}{DF})$

What do you notice?

13) PRACTICE: Find $x$ in each of the diagrams below.
14) Extension

a) What do you know about an angle that has a tangent ratio greater than 1? Less than 1? Equal to 1?

b) Give a possible perimeter for a triangle that has a tangent ratio of 0.75.

c) Why is there no sine or cosine ratio greater than one?
Practice Trigonometric Ratios

Give the ratios for the sine, cosine and tangent for the angles.

1) Sin A = _____  
   Cos A = _____  
   Tan A = _____

2) Sin B = _____  
   Cos B = _____  
   Tan B = _____

Using the Trig Table or a calculator, find the missing side.

3) Angle A = 22°

4) Angle 42°

5) Angle 34°

6) Angle 52°
Using the Trig Table or a calculator, find the missing angle.

8) Find **Angle** A

\[
\begin{align*}
\text{A} & \quad 4 \quad 7 \\
\end{align*}
\]

9) Find **Angle** D

\[
\begin{align*}
\text{D} & \quad 12 \quad 20 \\
\end{align*}
\]

10) Find **Angle** M

\[
\begin{align*}
\text{M} & \quad 8 \quad 16 \\
\end{align*}
\]

11) Find **Angle** R

\[
\begin{align*}
\text{R} & \quad 3 \quad 12.1 \\
\end{align*}
\]

12) For the regular pentagon, find TD. CE = 8 (Hint: What is \( \frac{1}{2} \) of angle CTE?)

\[
\begin{align*}
\text{C} & \quad \text{D} \quad \text{E} \\
\text{T} & \quad \text{D} \\
\end{align*}
\]
HOW HIGH? Surveyor’s Trig Trick

In this lesson, students will measure the heights of various objects using angle measurement and trigonometry. If students can measure how far they are from an object and the angle of inclination of their line of sight, they can use the tangent function to determine the height of the object at which they are looking. For example, assume the student has an eye height of 65” and is standing 17 feet (204”) from a lamp post looking up at the top of the lamppost at a 25 degree angle. The total height, T, is the sum of the preliminary height, P, (from the eye up) and the eye height, E, (from the eye down).

\[ \tan(25) = \frac{P}{204} \]
\[ 0.46 = \frac{P}{204} \]
\[ P = 0.46 \times 204 \]
\[ P = 95 \]

\[ T = P + E \]
\[ T = 95 + 65 \]
\[ T = 150” (12.5 feet) \]

LESSON PLAN
1. Introduce the project and describe the protractor/straw tool used. It is best to have an example to show the students. This is a crude model of an actual instrument, thus the measurements are approximations only.

2. Review the tangent function with a diagram on the board. Emphasize that the tangent of an angle is the ratio of the opposite side and the adjacent side. Show how you can solve for the height (the opposite side) if you know the horizontal distance (adjacent side) and the angle. They will be doing this for the lesson.

3. It is helpful to do a brief example of a height measurement in class to show the students. Measure the height of a door or wall with one of the tools. Show the calculations necessary to determine the height.

4. They are now ready to measure the objects. Go to the first object and let the students begin measuring. Remind them that they can measure from any distance. You can even ask the students to measure the same object from several distances to show that the height does not change.

5. While they measure, suggest to the students that one of them should look through the straw while the other keeps the protractor level, and others in the group may then measure the distance.

6. During the debriefing portion of the activity, emphasize to students that the smaller angle generated a smaller ratio, because the horizontal distance (the denominator) is larger (further away from the wall) at the smaller angles.

7. Also stress the meaning of the decimal version of the ratio. For example, 0.75 means that the vertical height is 75% of the horizontal distance.

This is the instrument used for this lesson:
HOW HIGH? Surveyor’s Trig Trick

Surveyors (those hard-working people in the orange vests along the roadside or at a construction site) often use trigonometry to measure unreachable distances. Similarly, you are to use the tangent function to calculate unreachable heights.

1. Measure the height of your eyes.
2. Look through the straw towards the top of the object.
3. Have your partner hold the protractor against the side of the straw. Be sure the straw passes through the center of the protractor. Be careful to keep the protractor level with the ground. Read and record the angle formed by the straw.
4. Measure the distance from you to the object. Use this distance and the tangent ratio for the measured angle to determine the height of the object.

A typical situation looks like this... The instrument for this lesson will look like this...

For each object above, on the backside of this sheet, draw and label the appropriate triangle diagram and show the corresponding calculation used to determine the height of the object. Be sure to show the measured angle and its tangent ratio.
Boot Camp: Special Right Triangles

Turn to page 755 in the textbook. Copy examples 1 and 3 at the bottom of this page. Then, simplify the problems below.

1) \(\sqrt{45}\)

2) \(\sqrt{196}\)

3) \(6\sqrt{200}\)

4) \(\frac{\sqrt{3}}{4\sqrt{5}}\)

5) \(\frac{\sqrt{3}}{5\sqrt{5}}\)

6) \(\frac{x}{4x\sqrt{2}}\)

7) \(\frac{\sqrt{4}}{x\sqrt{3}}\)

8) \(\frac{3\sqrt{4}}{\sqrt{3}}\)
Turn to page 425. Read and copy Theorems 8-5 and 8-6 below. Carefully read examples 1-4, taking notes as you deem necessary. Once complete, find the missing side lengths. Leave your answers as radicals in simplest form.

9)

10)

11)

12)

13)

14)

15)

16)
Ratio-a-Ratio
Trigonometry meets Special Right Triangles

Let’s explore the relationship between the trigonometric ratios and the special right triangle ratios. Begin by finding the lengths of the sides for each set of triangles. Within each set, the first triangle is given in terms of $x$, the second is an instance with one side given, while in the third you are expected to create an instance. Then complete the charts according to the values of the triangles. Offer both fraction and decimal values. Be sure to RATIONALIZE all denominators.

45-45-90

Ultimate Cosmic Example  Instance 1  Instance 2

Tan $45^\circ$ \[ \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opposite}}{\text{adjacent}} \]

sin $45^\circ$ \[ \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opposite}}{\text{hypotenuse}} \approx \frac{\text{opposite}}{\text{hypotenuse}} \approx \frac{\text{opposite}}{\text{hypotenuse}} \approx \frac{\text{opposite}}{\text{hypotenuse}} \]

cos $45^\circ$ \[ \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{\text{adjacent}}{\text{hypotenuse}} \]
Use your exact values to determine the other two sides of each triangle. Do these values support your special right triangle ratios?

### Ultimate Cosmic Example

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<th>Instance 1</th>
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<td>sin 30</td>
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### Instance 1

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### Instance 2

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### Compare to the calculator’s functions

**Exact values according to YOU**

- Tan45
- Sin45
- Cos45

**Approximate values according to the CALCULATOR FUNCTIONS**

- Tan45
- Sin45
- Cos45

**Assignment**

Use your exact values to determine the other two sides of each triangle. Do these values support your special right triangle ratios?
1. You want to build a bridge across a lake but can’t walk across the lake and measure it directly. So you measure the following distances and angles. How long will the bridge be (to the nearest foot)?

2. There is a pole with a guy wire attached to the top and anchored at a sixty degree angle with the ground ten feet from the pole (as shown below left). Then another wire is anchored to form a 45° angle and a third to form a 30° angle. How far from the pole are each of the other two wires anchored?

3. Angle A has a larger tangent ratio than angle B. Which angle is larger? Justify your answer by any means other than citing instances from your calculator.

4. Suppose the owner of the factory needs to install a new ramp for the loading dock. The ramp makes a 5° angle with the ground. How far will this ramp extend from the loading dock? Explain.

5. The hypotenuse of a right triangle measures 9 inches, and one of the acute angles measures 36°. What is the area of the triangle? (Round to the nearest square inch.)
6. Duffy thinks that since sine involves the \textit{opposite} side, and cosine involves the \textit{adjacent} side, and that tangent is defined as the ratio of \textit{opposite} to the \textit{adjacent} side, that the tangent of an angle will equal the sine divided by the cosine of the angle. Mathematically support or refute his claim below.

\[
\tan A = \frac{\sin A}{\cos A}
\]

7. Given the three points B(-2, -4), C(3, 3), D(-2, 3), find all three angle measures of \(\triangle BCD\).

8. a) Show that the slope of the line below is equivalent to the tangent of the angle formed with the x-axis.

b) Is this true for all lines?

9. The dimensions of the following picture are given. If the true cow is 84” long, how tall is it?

\[
\begin{array}{c}
\text{1.25 in} \\
\text{.75 in} \\
\end{array}
\]

10. Given that \(\triangle PQR \sim \triangle PST\), find the scale factor and the coordinates of S.

11. Find the lengths of the legs in the triangle below (rounded to the nearest whole unit). Use the Pythagorean theorem to confirm your answer.
Quiz: Trigonometric Ratios

1. Find the tangent ratio of angle P.

2-4) Use trigonometric ratios to solve for missing sides and angles.

2.

3.

4.

5. Jorge used a straw and protractor to measure the height of the lamppost, as shown below. While standing 204” from the post, he was looking up at an angle of 25°. Given that his eye height is 65”, what is the height of the lamppost?

6. Show why the exact value of $\sin 45^\circ$ is $\frac{\sqrt{2}}{2}$. 
Quiz: Trigonometric Ratios

7. Which triangle, ABC or ABC’ has a larger cosine ratio for A? Why?

8. Given the three points B(-2, -4), C(3, 3), D(-2, 3), find the measure of angle D.

9. The dimensions of the following picture are given. If the true sign is 5’ tall, how wide is it?

10. What is the perimeter of the triangle?
1. Show why the exact value of …
   a) \( \sin 60 = \frac{\sqrt{3}}{2} \)
   b) \( \cos 45 = \frac{\sqrt{2}}{2} \)
   c) \( \tan 30 = \frac{\sqrt{3}}{3} \)

2. As the length of BC* gets longer, does the cosine of A get larger or smaller? Justify your answer.

3. Jorge used a straw and protractor to measure the height of the lamppost, as shown below. While standing 178” from the post, he was looking up at an angle of 27°. Given that his eye height is 62”, what is the height of the lamppost?
4. It is said that \( \sin^2 x + \cos^2 x = 1 \)

Use the exact values of the instances below to support or refute this statement.

<table>
<thead>
<tr>
<th>x</th>
<th>sin</th>
<th>cos</th>
<th>( \sin^2 x )</th>
<th>( \cos^2 x )</th>
<th>( \sin^2 x + \cos^2 x )</th>
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</thead>
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<td>30°</td>
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<td>45°</td>
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<td>60°</td>
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Use the approximate values of three more instances to justify your claim above.

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<th>sin</th>
<th>cos</th>
<th>( \sin^2 x )</th>
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Use the diagram below to prove \( \sin^2 x + \cos^2 x = 1 \)

\[ \text{Diagram showing a right-angled triangle with sides } a, b, 	ext{ and hypotenuse } c. \]