Shipping Conundrum: The Pythagorean Theorem in 3-D

You have been asked to ship a large broom to your scary Aunt Matilda in Transylvania. The only problem is that the broom is 8 feet long. (Why does she need an eight foot broom, anyway?!?)

The post office will ship any box as long as the length and the width (the two shortest sides) do not add up to more than 108 inches. The third side can be any length. After talking to the people at the postal annex, you find out that there are four different boxes that might be large enough to work. Their measurements are listed below:

#1: 4’2” x 5’ x 5’10”    #2: 3’4” x 5’10” x 3’4”    #3: 2’6” x 5’10” x 5’10”    #4: 7’6” x 10” x 10”

Before determining which box is best suited to ship Auntie M’s broom, prove the conjecture below. Once you are confident that the conjecture is true, use it to show which boxes will be big enough to hold the broom:

<table>
<thead>
<tr>
<th>Box #1</th>
<th>Box #2</th>
<th>Box #3</th>
<th>Box #4</th>
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</table>

Of the boxes that are big enough to hold the broom, which is small enough to ship? Why?

Conjecture: Given a rectangular box with dimensions of x, y & z, the square of the length of the diagonal of the box will be equal to the sum of the squares of the dimensions of the box. In other words, \( d^2 = x^2 + y^2 + z^2 \).

Prove the conjecture:

Verify the conjecture: Use two different instances to support your proof.
(Choose two sets of three numbers and show that the formula is true.)

Instance #1           Instance #2