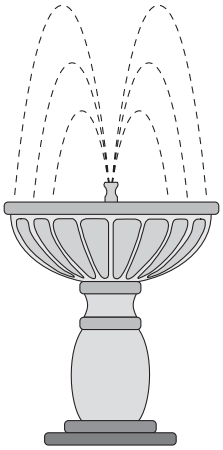


THE COIN FOUNTAIN

The study of parabolic curves through the design of water arcs

LESSON PLAN

The twelve questions on the student handout are listed in ascending order of difficulty. The number of elements that students will be capable of completing depends on the course. We suggest the following:

Beginning Algebra: #1-7
Advanced Algebra: #1-10
Bonus/Revisit: #11-12

The overall purpose of this lesson is to understand the relationship between an equation and its roots. Rather than deconstructing an equation to get to the roots, this lesson constructs the equation from the roots.

Present the students with the project at the beginning of a unit on quadratic equations. Over several days, take the entire class through each phase of the activity using the solution offered below: launch (0, 1), land (0, 17) and vertex (9, 32). These elements offer a very easy value of "a." While, students usually have little difficulty with Part One, the challenges come in Part Two, where the students find the roots by several methods. The fact that the students repeatedly achieve their launch and landing points as solutions reinforces their understanding of the roots.

After completing the assignment with the given values, the students are to complete the assignment again using their own values. They choose the launch and land points of the arc, and write an equation that produces a height within the given range. The purpose for this range of acceptable heights is to force the students to "play" with a variety of values for "a" until they get a correct one. This better helps them understand the influence of "a" on the graph, rather than if the students simply solved for "a."

Concepts

Writing, graphing and solving quadratic equations; finding the roots, vertex, axis of symmetry, directrix, and focus; solving by factoring, completing the square, and the quadratic formula; systems of quadratics

Time: 2-4 hours

Materials

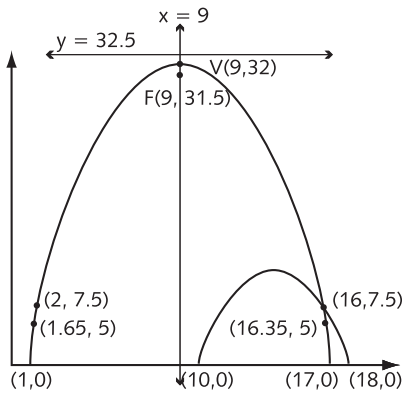
Graph paper, calculator and the student handout

Preparation

Students need to have a basic understanding of each of the methods at each step. This project gives them a conceptual understanding of what the methods produce for them, but it is not an exploratory or investigative activity.

SAMPLE SOLUTIONS

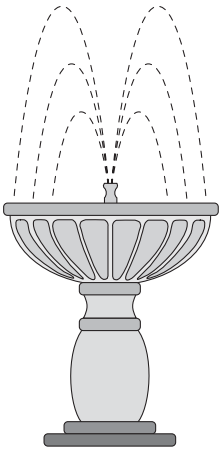
1.



2. $y = a(x - 1)(x - 17)$
   $y = a(9 - 1)(9 - 17)$
   $y = a(-8)(8)$
   $y = -64a$
   i.e. $a = -0.5$ produces height of 32
   $y = -0.5(x - 1)(x - 17)$
   $y = -0.5x^2 + 9x - 8.5$
3. Roots (1,0) & (17,0) for height of 32
4. $y = -0.5(2 - 1)(2 - 17) = 7.5$
   The arc will be 7.5 ft high after 1 ft.
5. vertex: V(9,32)
   axis: $x = 9$
6. $y = -0.5x^2 + 9x - 8.5$
   $y = -0.5(x^2 - 18x + 17)$
   $y = -0.5(x - 1)(x - 17)$
   $x - 1 = 0$ or $x - 17 = 0$
   $x = 1, 17$
7. $(-9 \pm \sqrt{9^2 - 4(-0.5)(-8.5)})/(2(-0.5))$
   $x = 1, 17$
8. $8.5 = -0.5x^2 + 9x$
   $8.5 = -0.5(x^2 - 18x)$
   $8.5 - 40.5 = -0.5(x^2 - 18x + 81)$
   $y = -0.5(x - 9)^2 + 32$
9. $5 = -0.5x^2 + 9x - 8.5$
   $0 = -0.5x^2 + 9x - 13.5$
   $(-9 \pm \sqrt{9^2 - 4(-.05)(-13.5)})/(2(-.05))$
   $x = 1.65, 16.35$
10. $50 = a(x - 1)(x - 17)$
    $50 = a(9 - 1)(9 - 17)$
    $50 = a(-8)(8)$
    $50 = -64a$
    $a = -(25/32)$
11. Second arc: $y = a(x - 10)(x - 18)$
    $10 = a(14 - 10)(14 - 18)$
    $a = -0.625$
    $y = -0.625x^2 + 17.5x - 112.5$
    $-0.5x^2+9x-8.5=-.625x^2+17.5x-112.5$
    $0.125x^2 - 8.5x + 104 = 0$
    $x = 16, 52$; (16, 7.5)
12. $1/(4a) = 1/(4(-0.5)) = -0.5$
    Focus: F(9,32 - 0.5) = F(9,31.5)
    Directrix: $y = 32.5$

THE COIN FOUNTAIN

The study of parabolic curves through the design of water arcs



You have been hired to design the water arc of a coin fountain. The pool of the fountain is 20 feet wide, and the water arc is to be greater than 6 feet tall, but less than 50 feet. You will need to determine the locations of the launch point and landing points and the maximum height of the arc. Also, you will need to write an equation that describes the water arc in terms of its height in relation to the horizontal distance along the pool.

PART ONE

1. Place the side view of your fountain in a first quadrant graph. Have the surface of the pool correspond to the x-axis with the left side at the origin. Show the coordinates of the roots and vertex, and include all pertinent data points from the following questions.

2. Using the equation $y = a(x - x_1)(x - x_2)$ where $(x_1, 0)$ & $(x_2, 0)$ are the roots of the parabola, choose a value for "a" that will produce a reasonable arc. Then convert your equation to the form: $y = ax^2 + bx + c$.

3. State your launch and landing points and the height of your water arc.

   Launch (,)
   Landing (,)
   Height _____

4. After one foot of horizontal distance (from your launch point) how high will the water arc be?

5. Find the vertex and the equation for the axis of symmetry.

   Vertex (,)
   Axis: x =

THE COIN FOUNTAIN

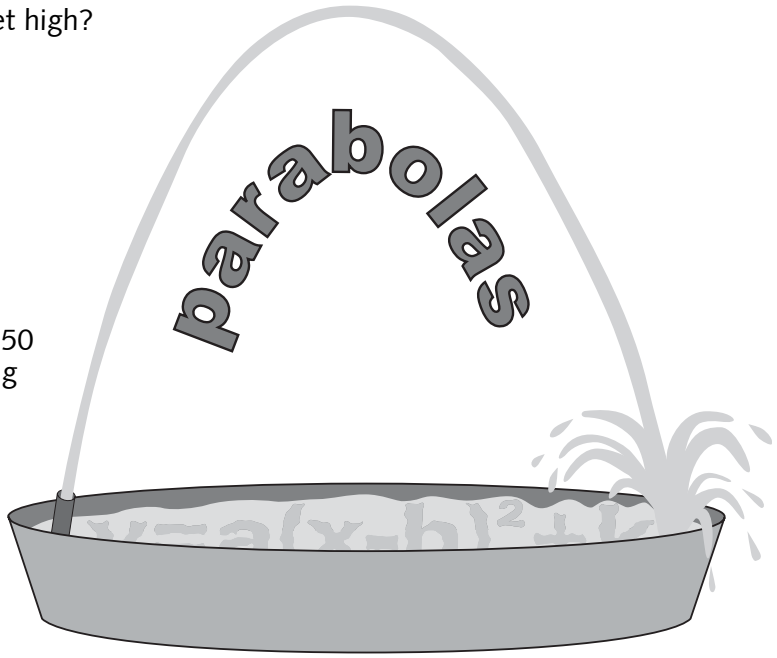
PART TWO

6. Factor your equation (in the form $y = ax^2 + bx + c$) to find the roots of the equation and then verify your launch and landing points.

7. Use the quadratic formula to find the roots, again verifying the launch and landing points.

8. Convert your original equation from quadratic form to vertex form, $y = a(x - h)^2 + k$, by completing the square.

9. At what horizontal distance will the arc be 5 feet high?



PART THREE

10. Determine the value of "a" that will produce a 50 foot high arc for your chosen launch and landing points.

11. Create a second arc that intersects the first one and find their point of intersection.

12. Create and graph the coordinates of the focus and the equation for the directrix of your water arc.