

## LESSON PLAN

The lesson is divided into three phases: 1) practicing the skills and algorithms used in the project, 2) taking the measurements at the shot put pit, and 3) calculating the radius, the degree measure of the arc and the arc length.

**PHASE ONE: Practicing the Principles:** On the first page of the handout, have students complete Part A: Dissecting a Circle (steps #1-8). The completed solution diagram is shown to the right. Since the students will be generating their own chords within the circle, their actual measurements will vary. The important points are that  $(AP)(PB) = (CP)(PD)$ , the radius is  $\frac{1}{2}(CD)$ , and the degree measure of the arc can be found by using trig functions. In step #8, the use of the string in measuring the actual length of the arc is important because this is what the students will actually do in the shot put pit.

In Part B: Completing a Circle (step #9), the students have an opportunity to test their new understanding of these principles. The solution diagram is given here. Assuming all the students measure in millimeters, their measurements should be very similar.

**PHASE TWO: Measuring the Pit:** The next phase of the project is to have students actually measure the various parts of the arc in the shot put pit. The first two measurements, the chord

and distance from the midpoint of the chord to the arc, are necessary for carrying-out other calculations later. The actual measuring of the radius and the arc are for verification purposes at the end of the project. It makes a very strong impression in the minds of students when their calculated answers are close to the measured answers.

**PHASE THREE: Calculating the Arc:** Once the measurements are acquired, the students should return to the classroom to make their calculations. The same process that they practiced on the first page of the worksheet should be implemented here, so limited guidelines are supplied to the students. In fact, the only two components that they are asked to record are the radius and the degree measure of the central angle/arc. From these two pieces of information, they can then calculate the length of the arc.

### Concepts

Circles, lengths of chords, perpendicular bisector of a chord, arc length, trigonometric functions

**Time:** 2-3 hours

### Materials

One ball of string (70 feet long), one compass and one tape measure for each group of 3-4 students; student handout.

### Preparation

Verify that the shot put pit at your school has the appropriate distance markings (arcs are typically spaced ten feet apart). If a shot put pit is not available, then any arc will do, like a curved set of stairs. Avoid conducting this project on a windy day, or your students' strings may end up a knotted mess.



# SHOT PUT ARC

Your task is to calculate the length of one of the arcs in the shot put pit. Before you do so, here are two exercises to study the principles that you will need to accomplish this task. You will be applying your knowledge of chords, trigonometry, and arc length. For these two exercises, make all measurements of length in millimeters.

## PART A: Dissecting a Circle

1. Draw chord AB anywhere in the circle such that it is NOT a diameter.

2. Draw a chord that is the perpendicular bisector of AB. Label the intersection of these chords point P and label the new chord CD so that CP is shorter than PD. Measure the following distances (in millimeters):

$$AP = \underline{\hspace{2cm}} \quad PB = \underline{\hspace{2cm}} \quad CP = \underline{\hspace{2cm}}$$

3. Use the formula  $(AP)(PB) = (CP)(PD)$  to calculate the length of PD. Then measure PD to verify your answer:

$$\text{Calculated Distance of PD} = \underline{\hspace{2cm}}$$

$$\text{Measured Distance of PD} = \underline{\hspace{2cm}}$$

4. Find the center of the circle and label it M. Draw in radii AM and BM. Calculate the radius of the circle, then measure it to verify your answer.

$$\text{Calculated Radius} = \underline{\hspace{2cm}}$$

$$\text{Measured Radius} = \underline{\hspace{2cm}}$$

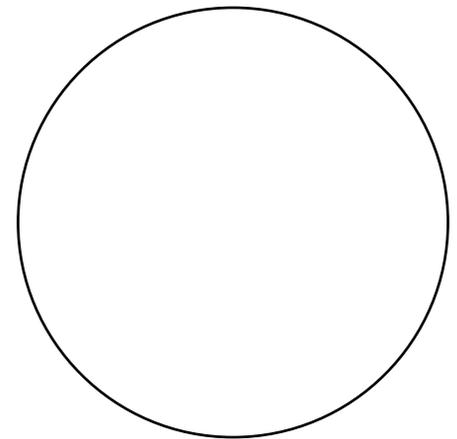
5. Verify your measurements for AP, PM, and AM by using the Pythagorean theorem.

6. Calculate  $m\angle AMP$  and  $m\angle AMB$ .

7. Use  $m\angle AMB$  and the radius to find the length of arc  $\widehat{AB}$ .  $\widehat{AB} = \underline{\hspace{2cm}}$

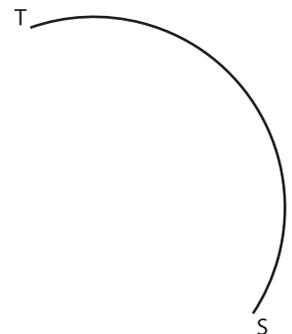
8. Measure the length of arc  $\widehat{AB}$  by laying a small piece of string atop the arc. Mark on the string, the length of the arc. Straighten the string and then measure the arc's length along the string. How accurate were you?

$$\widehat{AB} = \underline{\hspace{2cm}}$$



## PART B: Completing a Circle

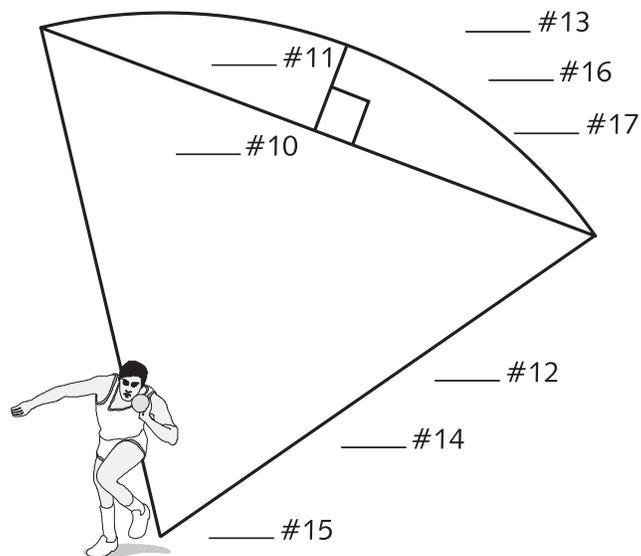
9. Use the algorithm above to find the length of arc  $\widehat{ST}$ , and to complete the circle.



# SHOT PUT ARC

## PART C: Measuring the Pit

10. With the string, form a chord from one end of your arc to the other. Measure the length of this chord.
11. Measure the distance from the midpoint of the chord to the arc, along the perpendicular bisector.
12. Measure the radius of your arc. Be sure to measure to the true center of the circle.
13. Now lay your string along the arc itself. Mark the length of the arc on the string itself. Straighten the string and then measure the length of the arc along the string.



## PART D: Calculating the Arc

- 14) Calculate the radius of the arc. It should be close to the the value that you measured, but not necessarily exact.
- 15) Calculate the degree measure of the arc.
- 16) Calculate the length of the arc.
- 17) Calculate your margin of error between your calculated length and the length that you measured in the pit.