PROJECT

Princess Dido: The Geometry of an Ox Skin

THE LEGEND OF PRINCESS DIDO

According to the epic *Aeneid*, Dido (pronounced "Dee Dough") was a Phoenician princess from the city of Tyre (now part of Lebanon). Her treacherous brother, the king, murdered her husband, so she fled the city and sailed with some of her loyal subjects to Carthage, a city on the northern coast of Africa. She wished to purchase some land from the local ruler in order to begin a new life. However, he didn't like the idea of selling land to a foreigner. In an attempt to be gracious and yet still spoil Princess Dido's request, the ruler said, "You may purchase as much land as you can enclose with the skin of an ox." Undaunted, Princess Dido and her subjects set about the task by slicing the ox skin into thin strips and then tying them together to form a long band of ox hide, and foiling the ruler's malicious plan. (Hildebrandt & Tromba, *Mathematics and Optimal Form*, 1985)

According to this legend, how large of an area could Princess Dido have feasibly enclosed with the ox skin?

The isoperimetric principle (for a given perimeter, the circle offers the greatest area) is the central mathematical idea that drives the ancient legend of Princess Dido as well as this lesson. However, since much of this lesson is a review of the major concepts in geometry it best serves as a culminating activity of the course.

Concepts

Area, perimeter, isoperimetric principle, fundamental theorem of similarity, measurement, unit conversion.

Time: 3-5 hours

Materials

One king size bed sheet, with lines drawn lengthwise 1 inch apart, class set of scissors & rulers, student handouts

Preparation

Prepare the bed sheet and its pillow case by drawing parallel lines one inch apart. Also choose a area, such as a football field, on which to cut the sheet on the final day of the project.

LESSON PLAN

THE PRACTICE: Randy Rat (2-3 Hours)

Share the legend of Princess Dido with the class while a king size bed sheet is laid out on the floor. After the story, tell the students that they will be cutting this bed sheet into one inch strips and finding how big of an area Princess Dido could have conceivably encompassed. Ask them to take a guess as to how large of an area the bed sheet could surround. Most students estimate it to be about the size of two classrooms.

The students will first investigate the elements of the Ox Skin challenge on Randy Rat. The imaginary rat skin is superimposed onto conventional graph paper, and is small enough to experiment with on a desk.

Part 1: Hopefully, all the students will breakdown the problem into sub-problems (the four rectangles representing the nose, head, torso and tail). From here, the students will implement one of two strategies: a) convert the dimensions into inches and then calculate the area or b) most commonly, they will find the area of the rectangles in square units, then convert to square inches. Many of those who must convert the area from square units will make the common error of dividing by 4 linear units instead 16 square units. Randy Rat's head is a square inch in which exactly 16 square units can be seen.

Part 2: Next, challenge the students to determine a cutting strategy that produces the longest half-inch wide rat skin strip. (There are actually many ways to cut, but only ONE final length). The rules for cutting are simple: The strips must be a half-inch wide (two units). Yes, two quarter-inch wide units may be set side by side. The intermediate strips must be rectangular (squared ends). In other words, when the students are done cutting they should form one long rectangle that is a half-inch wide. (This is the key which you do not want to reveal until after the students have investigated it for themselves: The strip is actually a rectangle with an area of 20 in^2 — conservation of area — and a width of one-half. The only possible length is 40). Have the students complete this in three distinct steps: PREDICT the length of the strip, CUT the strip laying the pieces end-to-end, and then MEASURE the strip to confirm the prediction. It is suggested that each group be given one extra copy of Randy Rat to cut up, and that all calculations be shown on the backside of their original copy.

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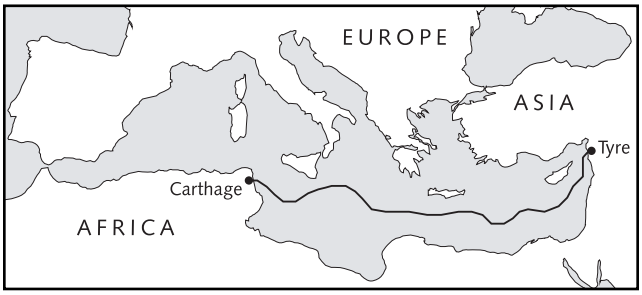
LESSON PLAN (continued)



Princess Dido's journey through the Mediterranean Sea

Part 3 : Once the students have discovered the answer of 40 inches, have them now choose three shapes (one triangle, one non-square quadrilateral, and any other shape such as a polygon circle or irregular figure). Each of the three should have a perimeter of 40. The students are to find the shape that yields the largest area (Princess Dido really wants to stick it to the king). The students will more than likely investigate isosceles triangles, rectangles that are close to squares, regular polygons (pentagons and octagons because the number of sides are factors of 40) and a circle. Some will try a right triangle, because of the ease of calculating the area. However, they will be challenged to find a Pythagorean triple that has a sum of 40 (8, 15, 17) and will eventually try an alternate strategy. Browse the room and choose several students to place one of their figures on the board. Have the triangles to the left with the number of sides increasing as you view the board until the circle is found to the far right. Then point out to the students how within each category of figures, the regular figure yields the greatest area. Also, the area increases as the number of sides increases until we see that **"for a given perimeter the circle yields the largest area" (isoperimetric principle)**. The students then can test their accuracy by forming a circle with the strips (standing on their edge) to form a circle, and measuring the diameter to confirm that it is indeed what they calculated (just under 13 inches).

Part 4: To later understand how Princess Dido could have encompassed an entire city with the ox skin, have the students revisit the idea that the ratio of the areas of two similar figures is the square of the ratio of their perimeters.

THE PREDICTION: Ollie Ox - The Bed Sheet (< 1 hour)
Once the students are familiar with the process that lies ahead with the ox skin (bed sheet), they will make the calculations on how big they believe the ox skin shape (circle) will be. A standard King Size bed sheet is about 8 x 9 feet with a pillow case of about 2 x 3.5 feet. Calculate only one side of the pillow, but allow the students to cut both to make up for any overlap. (Yes, we fudge here a little.) Depending on the exact dimensions of the bed sheet, the students will calculate a circumference of over 11,000 inches. Demand that they put this in terms that are commonly understood. The students should get an area of about 70,000 square feet (1.5 acres) and a circumference of 1,000 feet, and a diameter of about 300 feet (the length of a football field!!).

THE PRACTICUM - Princess Dido (1-2 Hours)
The climax for the students will be the actual cutting of the bed sheet to test their calculations. Ahead of time, determine which large area on your campus you wish to enclose with the strand (e.g. gym, administration, or the football field). The actual cutting is the longest phase. It is suggested that you have two students on opposite sides of the sheet cut the sheet in two, then add two more students to cut each half in half, and so on. Soon many will be cutting at the same time. Assign a few people to be staplers, being sure that they staple as shown in the diagram given them. Also have runners, who deliver the strips from the cutters to the staplers. Have the digital camera ready when they are all done!! This will take a little more than one hour, so if possible, seek permission to combine two classes into a block for this day only.

Use the last half hour to debrief the mathematics involved and explore what Princess Dido could have accomplished with strips that are thinner than those used by the students. Hildebrandt & Tromba claim that the semi-circle wall around Carthage (which is a port city along the coast, way to go Dido) encloses approximately 60 acres. That would require cutting the bed sheet into strips that are between a quarter and an eighth of an inch wide...or about the width of a rawhide shoelace. Could the legend be true?

Princess Dido: The Geometry of an Ox Skin

PART ONE: The Puzzle
How large of an area do you believe that Princess Dido could have feasibly enclosed with the skin of an ox?

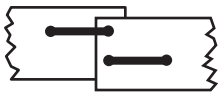
PART TWO: The Practice
Use Randy Rat to familiarize yourself with the process involved in approximating the possible area. Answer Questions 1-8 on the handout.

PART THREE: The Prediction
Using the actual dimensions of the ox skin (bed sheet), calculate the perimeter, area and width of the shape made by cutting the ox skin into one-inch wide strips.

Dimensions of the bed sheet: _____ x _____ Dimensions of the pillow case: _____ x _____

Perimeter: _____ Width: _____ Area: _____

PART FOUR: The Practicum
Cut an ox skin sized sheet into 1 inch strips. Staple the ends as shown. Once completed, measure the width and perimeter of the enclosed area, and calculate the solution to Princess Dido's puzzle.



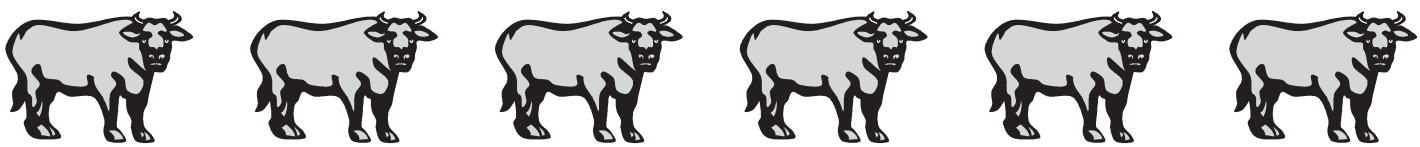
Perimeter: _____ Width: _____ Area: _____

Compare your results with your predictions and explain possible reasons for any discrepancies.

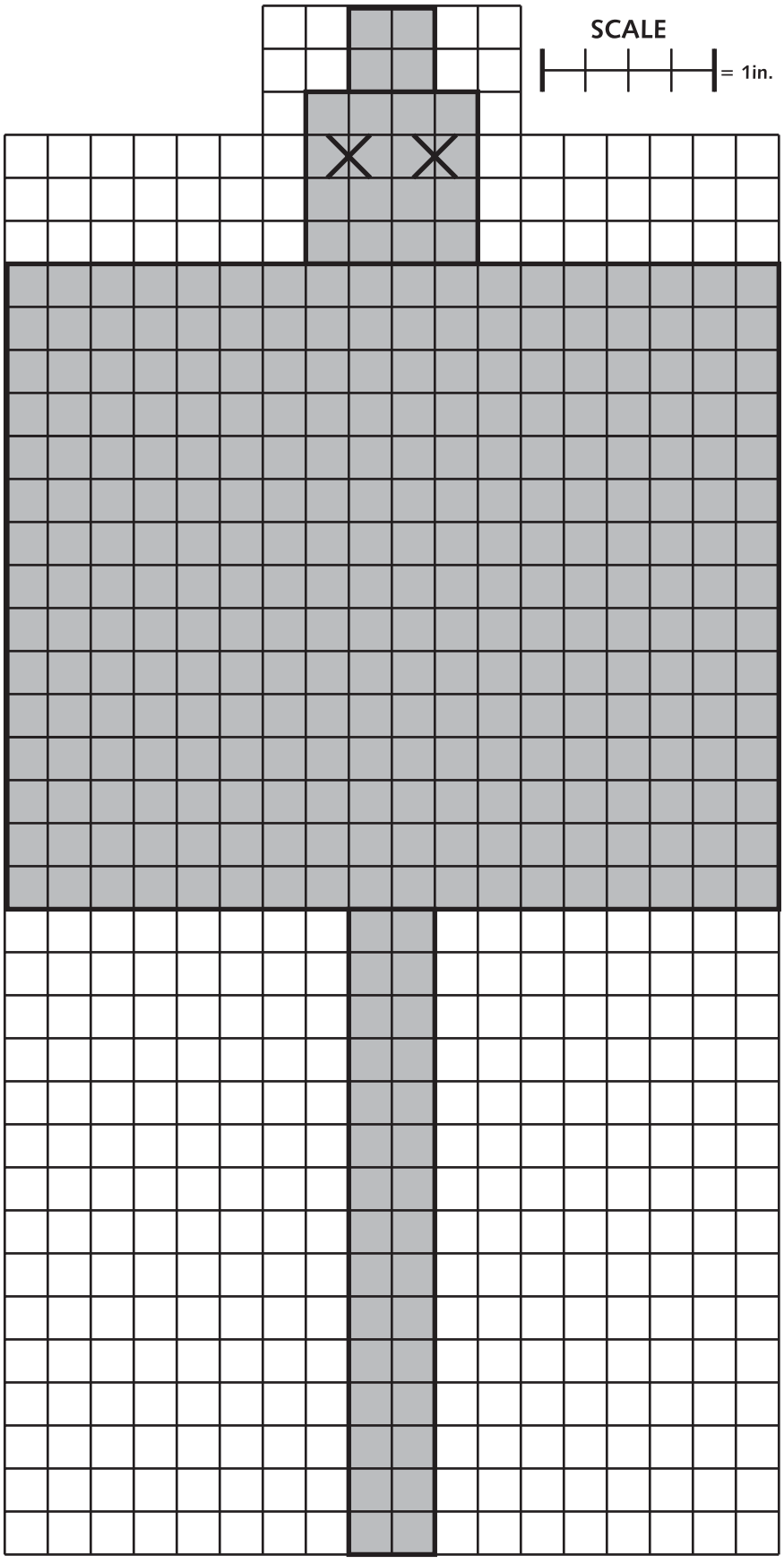
EXTRA CHALLENGE: What would the perimeter, area and width be for 1/2, 1/4 and 1/8 inch strips?

SUPER CHALLENGE: The *Aeneid* claims that Dido connected the two ends of the strand to a straight beach along the Mediterranean. If so, how much land could Dido have encompassed with the ox skin?

SUPER DUPER CHALLENGE: Hildebrandt & Tromba claim that Dido enclosed over 60 acres. If so, how thin would the strips needed to have been? (1 acre = 43,560 sq. ft.)



Randy Rat



1. What is the area of the rat skin? Use drawings and calculations of the sub-problems to show your thinking.

   Area = _____ in^2

2. If you were to cut the rat skin into 1/2 inch strips, what would be the length of the rat skin band, barring any overlap? (Hint: visualize the band as a long rectangle, 1/2 inch wide.)

   Length = _____ in

3. Choose three different shapes and use the perimeter from #2 to determine the area of each. At least one should have more than 4 sides.

   a. Which of your three shapes yields the largest area?

      _____

   b. What shape do you think will give the largest area for any given perimeter? If you have not done so already, calculate the perimeter and area for the optimal shape.

      A _____ with a

      perimeter of _____ and an

      area of _____

4. Assume that you cut the rat skin into $^1/_4$ inch strips. Calculate the perimeter and area of one of your shapes.

   a. What is the ratio of the perimeters between the two similar shapes?

      P_1/P_2 = _____ A_1/A_2 = _____

   b. What is the ratio of the areas between the two similar shapes?

   c. How do these numbers compare?