

The Gum Drop

Watch Your Step!

LESSON PLAN

The objective of the lesson is two-fold. The primary objective is to give students a general understanding of area. The use of a grid reinforces the idea that area is measured in square units. The size and number of these units may change, but the area of a given figure stays constant. The secondary objective is to introduce students to Pick's Algorithm.

PICK'S ALGORITHM (for finding the area of an irregular figure)

Set the figure on a grid. Coordinates are not necessary. The area of the figure will be the area of one grid unit times the sum of the interior grid squares (units contained entirely within the figure) and half the number of boundary grid squares (units that are only partially contained within the figure). This is based on the assumption that some of the grid squares will only have a small portion within the figure, while others will have a large portion, averaging approximately half of a grid square. The number of boundary squares can be increased by decreasing the size of the grid units. As the number of boundary squares increases, approaching infinity, the average gets infinitely closer to half. If we let N equal the number of interior squares, B equal the number of boundary squares and u equal one linear unit, then the area can be represented by the following formula:

$$A = (N + B/2)u^2.$$

Concepts

General concept of area, Area of Irregular figures (Pick's Algorithm), Fundamental Theorem of Similarity.

Time: 1-2 Hours

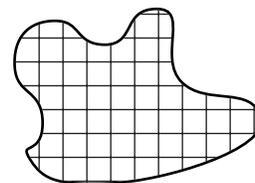
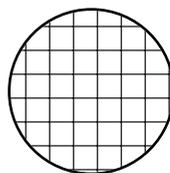
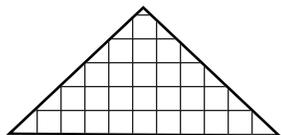
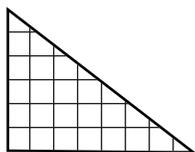
Materials

Student handout, paper, scissors.

Preparation

Find an area on campus that will contain a lot of gum drops (irregular black spots on the ground).

- 1) Have students find the area of the following first three figures by the traditional formulaic methods. Then ask them to count the number of grid squares to double-check. Students may show an intuitive sense of Pick's Algorithm by counting every two boundary squares as one unit.

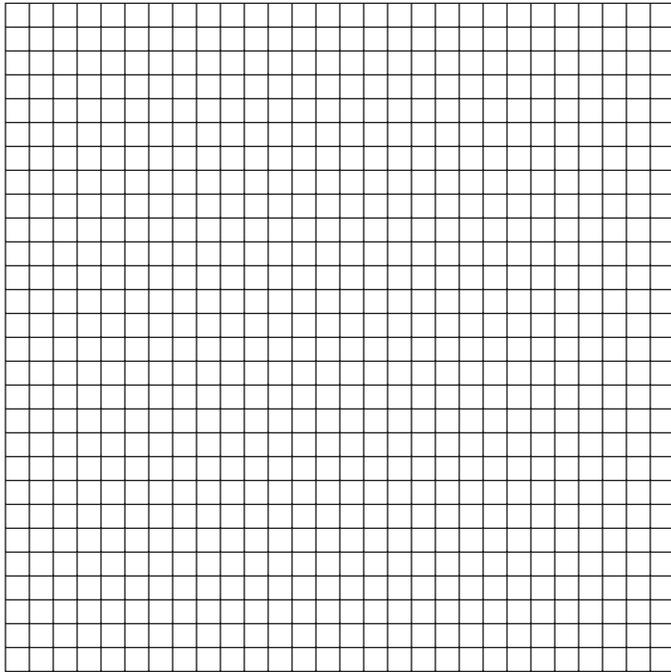


- 2) Have students practice their conjecture (Pick's Algorithm) on the fourth figure above. Emphasize the fact that the value of a square unit differs from the value of its side length (linear unit).
- 3) Seek out and harvest the gum drops. Have students find a black spot on the ground. Bigger is better. Encourage them to seek irregular figures as opposed to circular ones. Have students trace the gum drop onto paper. When they get back to class, they cut out these templates and trace them onto each of the three grids. This tracing procedure ensures that the figures on each grid will be congruent.



- 4) Have students calculate the area of each figure. While the procedure is the same for each one — count the interior units, divide the boundary units by 2, add these values and multiply by the value of one square unit — the students need to be made aware that the value of the square unit is equal to the area of the unit and not the length of the side. In other words, the area is $\frac{1}{4}$ in² for the $\frac{1}{2}$ inch grid, $\frac{1}{16}$ in² for the $\frac{1}{4}$ inch grid, and $\frac{1}{64}$ in² for the $\frac{1}{8}$ inch grid. If done correctly, the students should notice that the areas that they calculate are nearly equivalent.

The Gum Drop

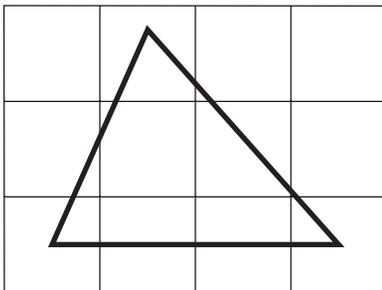


$\frac{1}{8}$ " GRID

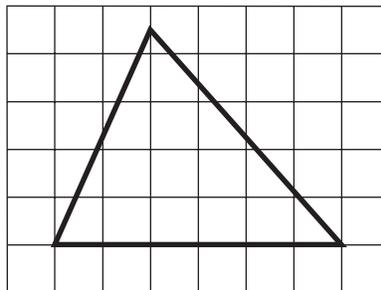
Area of the Gum Drop = _____ in²

Which grid do think offered the most accuracy and why?

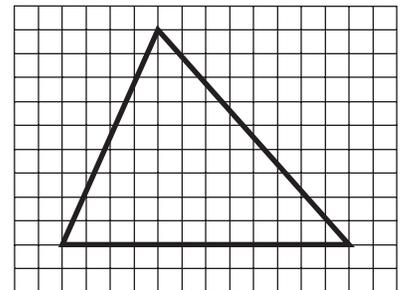
Test Pick's Algorithm again on the following triangle. For each grid, calculate the area of the triangle by the traditional formula, then apply your new method.



$\frac{1}{2}$ " Grid



$\frac{1}{4}$ " Grid



$\frac{1}{8}$ " Grid

By the Formula:

By Pick's Algorithm:

